# Power Performance Prediction for Nonlinear Waves in Shallow Water

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## Abstract

This paper looked at the power performance of wave with nonlinear characteristics in shallow water using wave theory of Stokes. The result showed that wave power was proportional to wave period for Stoke's theory, both for wave height and wave depth varied.

**Keywords:** Renewable energy, Wave energy, Nonlinear wave

## 1. Introduction

During the 1970s, it became clear that the world's known nonrenewable energy resources are decreasing rapidly and may be exhausted within the foreseeable future. In response to this disturbing prospect, the focus on building the renewable resources has begun. Ocean wave energy is one of those renewable energies that have been studied and researched.

The most conspicuous form of ocean energy is the surface wave. Waves are simply energy in transition, that is energy being carries away from its origin. One of the sources of wave energy is the "wind waves". Wind waves are actually a form of solar energy since the primary source of wind energy is the sun. Solar radiation is then collected by water.

It is most important to note that the distinction between deep and shallow water waves has little to do with absolute water depth but is determined by the ratio of water depth to wave length. In shallow water that ratio is less than or equal to 0.02.

In shallow water, wave becomes sharp crested with a broad trough; that is, the wave profile is nonlinear.

This purpose of this paper is intended to provide the theoretical analysis of wave in shallow water when nonlinear theories have to be applied, especially look into the power that wave can provide using the theory of Stokes.

## 2. Properties of Nonlinear wave

It should be noted that there is no mathematical theory that exactly describes the behaviour of water waves. The various wave theories simply approximate, to some degree, the actual phenomena.

Nonlinear waves have been found to occur in various physical disciplines and life science, manifesting phenomena that have been regarded, by and large, as being remarkable and often very challenging.

One failing of the linear wave theory is that it always predicts a sinusoidal profile. A deep water swell having a low value of will be well approximated by this profile; however, as the wave begin to shoal, that is, to be affected by the seafloor, the wave profile will begin to change to one with a narrow crest and broad trough. This profile is said to be nonlinear.

Still Water Level (SWL) and Mean Water Level (MWL) coincide for the linear wave, however, the SWL will be below the MWL for the nonlinear wave since the MWL is defined as being half the distance from trough to crest. Since the position of the MWL is defined by the wave height , this is the most logical level on which to place the origin of our coordinate system. [1]

### 2.1 Power of Nonlinear waves

In this section, we look at the nonlinear wave theorie, stokes's second order theory, concentrating on power, in a shallow water condition.

Stokes (1847, 1880) introduces an irrotational water wave theory that utilizes series representations of wave properties. The accuracy of the theory depends on the number of terms contained in the series. For example, Stokes' first order theory is identical with the linear theory. Stokes' second order theory improves the accuracy in determining the wave profile and, in addition, the mass transport convection velocity and the breaking condition of the wave. Stokes' third and higher order theories successively add to the accuracy of the wave profile prediction. (2) For our purposes, the second order theory is satisfactory. [1]

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Without derivation, the shallow water wave profile predicted by stokes' second order theory is obtained from

$$\eta = \frac{H}{2} cox(kx - \omega t) + \frac{3}{16}\frac{H^2}{k^2h^3}\cos[2(kx - \omega t)] \quad (1)$$

Where $\boldsymbol{k}$ is the wave number of equation (2)

$$k = \frac{2\pi}{\lambda} \quad (2)$$

and $\omega$ is the circular frequency $2\pi f$.

$$\eta = \frac{H}{2}\cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right) \quad (3)$$

By comparing equation (3) and (1), it can be seen that the first term on the right-hand side of equation (1) is the expression for the linear profile; thus the second term is simply a correction to the first order (linear) theory. The expression for the wavelength $\lambda$ and the phase velocity or the celerity are the same as those predicted by the linear theory, that is, expressions found in equations (4) and (5), respectively.

$$\lambda = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi h}{\lambda}\right) \quad (4)$$

$$c = \frac{\lambda}{T} = \frac{gT}{2\pi}\tanh(kh) \quad (5)$$

The expression for the horizontal water particle velocity in shallow water according to the linear theory is

$$u = \frac{\omega H}{2kH}\cos(kx - \omega t) \quad (6)$$

Whereas that according to Stokes's second order theory is

$$u = \frac{\omega H}{2kh}\cos(kx - \omega t) + \frac{3}{16}\frac{\omega H^2}{k^3h^4}\cos[2(kx - \omega t)] \quad (7)$$

Surface particle velocity actually increases as the depth decreases and wave's phase velocity decreases with decreasing depth, then at some point the maximum horizontal velocity of the surface particles will equal the phase velocity, that is

$$u_{\max}\Big|_{z=H/2} = c \quad (8)$$

A wave is said to break when the particle velocity at a crest and the phase velocity are equal; that is

At a crest, one can assume that

$$\cos(kx - \omega t) = \cos[2(kx - \omega t)] = 1$$

For shallow water, phase velocity is

$$c = \sqrt{gh} \quad (9)$$

Thus if the results of equations (9) and (6) are used, the breaking condition in shallow water according to the linear theory is

$$\frac{\omega H_b}{2kh} = \sqrt{gh}$$

or, incorporating the results of the equation (10), (wavelength in shallow water)

$$\lambda = \sqrt{gh}T \tag{10}$$

the breaking height is

$$H_b = 2h \tag{11}$$

Combing equation (9), (8), and (7) results in the breaking condition according to Stokes' second order theory. The result is

$$\frac{\omega H_b}{2kh}\left[1+\frac{3}{8}\frac{H_b}{k^2h^3}\right]=\sqrt{gh}$$

or, using the result of equation (10), the breaking height is

$$H_b = \frac{16\pi^2h^2}{3gT^2}\left[-1+\sqrt{1+\frac{3gT^2}{4\pi^2h}}\right] \tag{12}$$

Thus  of equation (12) depends on both the period and the depth in shallow water, whereas the breaking height of equation (11) is independent of wave period.

The expressions for the total wave energy and the wave power in shallow water obtained using Stokes' second order theory are, respectively,

$$E = \frac{\rho g H^2 \lambda b}{8}\left[1+\frac{9}{64}\frac{H^2}{k^4h^6}\right] \tag{13}$$

and

$$P = \frac{\rho g H^2\sqrt{gh}b}{8}\left[1+\frac{9}{64}\frac{H^2(gh)^2T^4}{(2\pi)^4h^6}\right] \tag{14}$$

Or

$$P = \frac{\rho g H^2\sqrt{ghb}}{8}\left[1+\frac{9}{64}\frac{H^2(gh)^2T^4}{(2\pi)^4h^6}\right]$$

where $\boldsymbol{b}$ is the crest width and the group velocity $\boldsymbol{c_g}$ is equal to the phase velocity  in shallow water, from equation (15).

In shallow water $\left(h \le \lambda/20\right)$, the waves remain stationary with respect to group boundaries; thus

$$c_g = c \tag{15}$$

The total energy for linear wave theory is obtained from

$$E = E_p + E_k = \frac{\rho g H^2 \lambda b}{8} \tag{16}$$

The transfer of wave energy from point to point in the direction of wave travel is characterized by the energy flux or, more commonly, wave power:

$$P = \frac{\rho g H^2 c_g b}{8} \tag{17}$$

By comparing equation (13) with equation (16) of linear theory, and equation (14) with equation (17) of linear theory, it can be seen that Stokes' second order theory simply adds a correction factor $\frac{9}{64}H^2/k^4h^6$ to the energy and power expression of the linear theory.

## 3. Numerical results

### 3.1 Relationship between Wave power and Wave period when Wave height varied.

A. Result

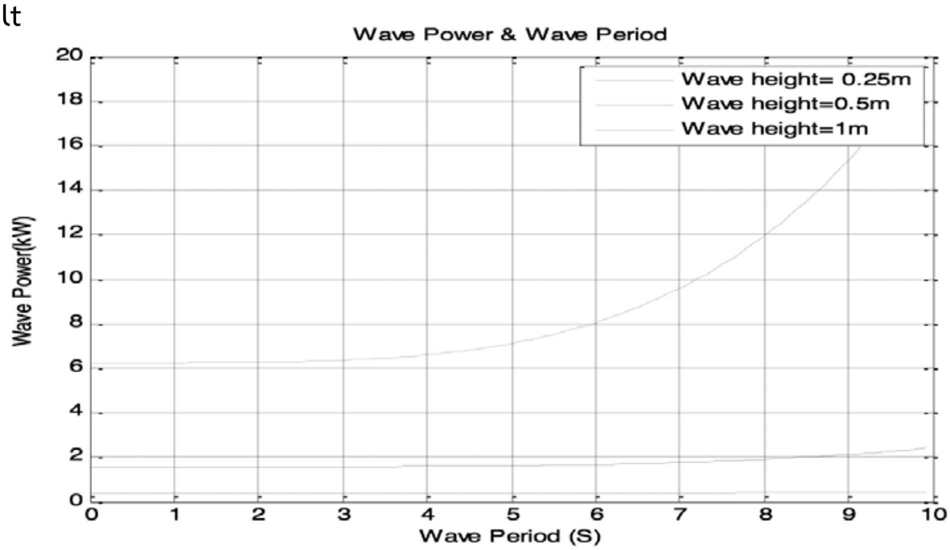


*Figure 1* *Graph Wave Power & Wave Period when Wave height varied for Stokes' Second order theory*

### 3.2 Relationship between Wave power and Wave period when Wave depth varied.

A. Result

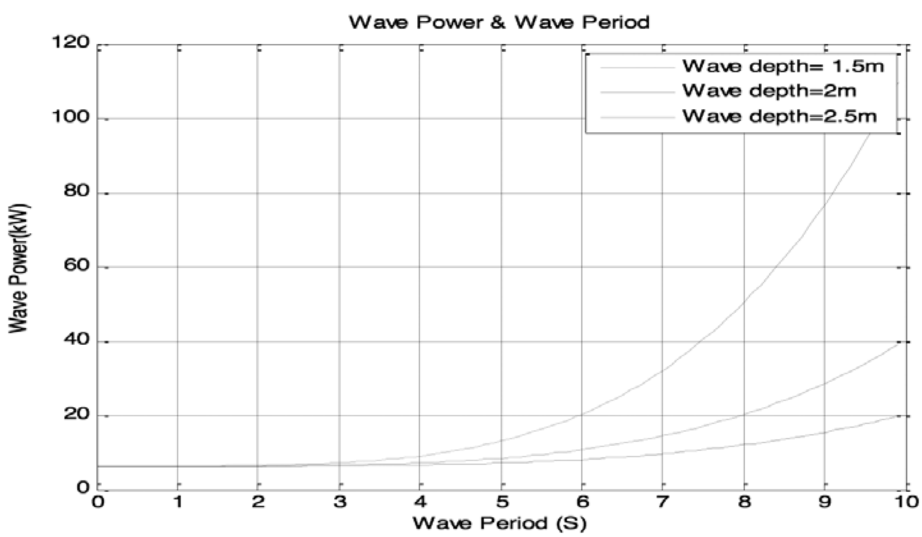


*Figure 2* *Graph Wave Power & Wave Period when Wave depth varied for Stokes' Second order theory*

## 4. Concluding Remarks

4.1 The Stokes' second order theory that analyzed in this paper were considered only in a shallow water, that is ratio of wave depth to wavelength is less than or equal 0.02.

4.2 The graph is plotted between wave power and wave period when wave height varied for Stokes' second order theory. The graph is shown in Figure 1. It can be seen that when wave period's increased, it made wave power increased. At the same wave period, if wave height's increased, it is increased the wave power.

4.3 Figure 2 showed graph of wave power and wave period when wave depth varied for Stokes' second order theory. It can be seen that when wave period's increased, it made wave power increased. At the same wave period, if wave depth's increased, it is increased the wave power.

4.4 Figure 1 and 2 yielded the power function graphs of roughly equation of $y=x^n+1$, when n=4. The reason they did not yield the proper power function graph because some constants involved which we can see from equation (14).

The reason we used Stokes' second order theory because its accuracy abilities to predict wave behavior in shallow water.

## References

[1] M. E. McCormick. 1981. Ocean Wave Energy Conversion: A Wiley-Interscience Publication,

[2] B. Count. 1980. Power from Sea Waves: Academic Press, Inc.,

[3] L. Debnath. 1990. Nonlinear Water Waves: Academic Press, Inc.,

[4] C. C. Mei. 1983. The Applied Dynamics of Ocean Surface Waves: A Wiley-Interscience Publication,

[5] P. G. Andres. 1991. Basic Mathematics for Engineers: John Wiley and Sons, Inc.

[6] C. H. Sisam 1990. Mathematics: A general Introduction: Hemry Holt and Company, Inc.